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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The investigators associated with the research reported herein have been supported by ARO almost continuously over the past 20 years, from the inception of their early development of penalty function and branch and bound methodology. This report covers results developed from 1 January 1974 to 31 May 1981 under two ARO contracts and one grant, results dealing mainly with nonconvex programming, second order algorithms and sensitivity and stability methodology. Technical details are omitted, the main intent being

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20. Abstract (Cont'd)
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FOREWARD

The investigators associated with the research reported herein have been supported by ARO almost continuously over the past 20 years, from the inception of their early development of penalty function and branch and bound methodology. This report covers results developed from 1 January 1974 to 31 May 1981 under two ARO contracts and one grant, results dealing mainly with nonconvex programming, second order algorithms and sensitivity and stability methodology. Technical details are omitted, the main intent being the provision of a concise chronicle of the major accomplishments.

Anthony V. Fiacco
Principal Investigator
August 1981

1. Problem Areas Studied

The following topics have been developed under the subject grant and contracts:

1) penalty-function methodology;

2) convergence and rate of convergence properties of NLP algorithms;

3) arc methods for NLP;

4) nonconvex programming techniques;

5) inexact and implicitly defined optimization;

6) NLP sensitivity and stability analysis.

The level of effort directed to the various topics was a function of preliminary theoretical breakthroughs and potential practical applications. The objective was always, not only to obtain new significant theoretical results, but also efficient and stable numerical algorithms, usable computer programs, and computational implementations that demonstrated practical applicability.

2. Summary of Key Results

Some of the most important results obtained in this research were:

1) one of the first rigorously developed NLP algorithms for moving along directed arcs and determining a local solution satisfying second-order necessary conditions;

2) new proofs of convergence and rate of convergence of conjugate gradient algorithms;

3) some of the earliest rigorous results in inexact linear programming;

4) one of the first general NLP step-size rules that yields second-order convergence of algorithms;

5) the adaptation of branch and bound methods of nonconvex programming methods for computing economic equilibria;

6) the provision of one of the first fundamental theoretical bases for the characterization of solution sensitivity (parameter derivative) information, for general classes of parametric nonlinear programming (NLP) problems, and the development of methods for approximating this sensitivity information using information generated by standard NLP algorithms;

7) the development of the first computer program (SENSUMT) interfacing solution and sensitivity analysis calculations, specifically, numerical techniques for calculating a solution and approximations of NLP problem parameter derivatives of the optimal solution value, optimal solution vector, and optimal Lagrange multipliers, using the well-known and widely used SUMT penalty function algorithm [1]*;

8) the accumulation of the first significant computational experience reported in NLP sensitivity analysis, and the demonstration via SENSUMT of the feasibility and practical applicability of detailed sensitivity analysis calculations and interpretations on practical problems including problems in inventory control [2], water pollution [3], [4], and structural design [5];

9) one of the first theoretical developments of sharp bounds on the optimal value directional derivative limit quotient for general differentiable parametric programs;

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*These numbers refer to the documents listed in the references, the associated results being spin-offs from the ARO subject grant and contracts, developed under other sources of support. All topics not so referenced were supported by ARO and are reported in the documents listed in Section 3 of this report.

10) the first comprehensive survey of basic NLP sensitivity and stability results, initiating the unification of this important collection of results;

11) an interface of sensitivity analysis results with other basic NLP algorithmic approaches, namely with reduced gradient, projected gradient, and separable programming methods;

12) approximation of optimal value differential stability results utilizing standard NLP penalty function algorithms, briefly reported in [7];

13) extension of optimal value differential stability results to inequality-equality constrained parametric problems and general optimal value continuity results;

14) introduction of a simple technique for obtaining computable bounds, for large parameter changes, on the optimal value of parametric NLP problems that are jointly convex in the decision variables and the parameters, implemented and applied to a water pollution problem, and reported in [6].

15) extension of the optimal value convex-bounding procedure to nonconvex parametric problems that are jointly separable or factorable, successful preliminary results being reported in [6] on several small problems; and

16) inclusion of the optimal value bounding procedure for jointly convex programs in the SENSUMT computer program.

3. Publications and Technical Reports

The publications and technical reports resulting from this research, listed in chronological order, are as follows:

McCormick, Garth P. and Klaus Ritter, 1974. Alternative proofs of the convergence properties of the conjugate-gradient method. Journal of Optimization Theory and Applications, Vol. 13, No. 5.

Falk, James E., 1974. Exact solutions of inexact linear programs, Institute for Management Science and Engineering Technical Memorandum Serial TM-64546, The George Washington University.

Falk, James E., Anthony V. Fiacco, and Garth P. McCormick, 1974. Optimal structural design and the general eigenproblem, Institute for Management Science and Engineering Technical Paper Serial T-302, The George Washington University.

McCormick, Garth P., 1974. An arc method for nonlinear programming, Institute for Management Science and Engineering Technical Paper Serial T-295, The George Washington University.

McCormick, Garth P., 1975. An arc method for nonlinear programming, SIAM Journal on Control, Vol. 13, No. 6.

Armacost, R. L., and Anthony V. Fiacco, 1975. Second-order parametric sensitivity analysis in NLP and estimates by penalty function methods, Technical Paper Serial T-324, Institute for Management Science and Engineering, The George Washington University.

McCormick, Garth P., 1976. Second-order convergence using a modified Armijo step-size rule for function minimization, Technical Paper Serial T-328, Institute for Management Science and Engineering.

Armacost, R. L., and Anthony V. Fiacco, 1976. NLP sensitivity analysis for RHS perturbations: A brief survey and second-order extensions, Technical Paper Serial T-334, Institute for Management Science and Engineering, The George Washington University.

Falk, James E., 1976. Exact solutions of inexact linear programs, Operations Research, Vol. 24, No. 4.

Bracken, J., and James E. Falk, 1976. Computation of particular economic equilibria, IDA Paper P-1208, Institute for Defense Analysis.

Fiacco, Anthony V., 1976. Sensitivity analysis for nonlinear programming using penalty methods, Mathematical Programming, Vol. 10, No. 3.

Armacost, R. L., and Anthony V. Fiacco, 1977. Exact sensitivity analysis using augmented Lagrangians, Technical Paper Serial T-349.

McCormick, Garth P., 1977. A modification of Armijo's step-size rule for negative curvature, Mathematical Programming, 13, 111-115.

Fiacco, Anthony V., 1978. Nonlinear programming sensitivity analysis results using strong second-order assumptions, Technical Paper Serial T-377, Institute for Management Science and Engineering, The George Washington University.

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Bard, Jonathan F., and James E. Falk, 1978. Computing equilibria via nonconvex programming, Technical Paper Serial T-386, Institute for Management Science and Engineering, The George Washington University.

Fiacco, Anthony V., and William P. Hutzler, 1979. Extensions of the Gauvin-Tolle optimal value differential stability results to General mathematical programs, Technical Paper Serial T-393, Institute for Management Science and Engineering, The George Washington University.

Fiacco, Anthony V., and William P. Hutzler, 1979. Basic results in the development of sensitivity and stability analysis in constrained mathematical programming, Technical Paper Serial T-407, Institute for Management Science and Engineering, The George Washington University.

Bracken, Jerome and James E. Falk, 1979. Computation of particular economic equilibria, Management Science 25, 697-703.

Fiacco, Anthony V., 1980. Nonlinear programming sensitivity analysis results using strong second order assumptions, Numerical Optimization of Dynamic Systems (L. C. W. Dixon and G. P. Szego, Eds.), North Holland.

Bard, Jonathan F., and James E. Falk, 1980. Computing equilibria via nonconvex programming, Naval Research Logistics Quarterly, Vol. 27, No. 2, 233-255.

Falk, James E., 1980. The methods of contracting ellipsoids, Technical Paper Serial T-418, Institute for Management Science and Engineering, The George Washington University.

McCormick, Garth P., 1980. Finding the Global Minimum of a function of one variable using the method of constant signed higher order derivatives, Technical Paper Serial T-411, Institute for Management Science and Engineering, The George Washington University.

Armacost, R. L., and Anthony V. Fiacco, 1980. Sensitivity analysis of a well-behaved Kuhn-Tucker triple, Survey of Mathematical Programming (Ed., A. Prékopa), North-Holland.

Fiacco, Anthony V., 1980. Continuity of the optimal value function under the Mangasarian-Fromovitz constraint qualification, Technical Paper Serial T-432, Institute for Management Science and Engineering, The George Washington University.

Fiacco, Anthony V., 1980. Optimal value differential stability bounds under the Mangasarian-Fromovitz constraint qualification, Technical Paper Serial T-435, Institute for Management Science and Engineering, The George Washington University.

Fiacco, Anthony V., and Abolfazl Ghaemi, 1980. A user's manual for SENSUMT, Technical Paper Serial T-434, Institute for Management Science and Engineering, The George Washington University.

McCormick, Garth P., 1980. Convergence theory for unconstrained minimization, Technical Paper Serial T-431, Institute for Management Science and Engineering, The George Washington University.

4. Participating Scientific Personnel

The Principal Investigator for the subject contract and grants was Anthony V. Fiacco and the Co-principal Investigators were Garth P. McCormick and James E. Falk. These constituted all the ARO-supported personnel, other participating individuals whose names appear as co-authors of various papers listed in Section 3 having other sources of support.

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